

Radiatively efficient disks around Kerr black holes

S. Campitiello^{1*}, G. Ghisellini², T. Sbarrato¹, G. Calderone³

¹*Dipartimento di Fisica “G. Occhialini”, Università di Milano – Bicocca, piazza della Scienza 3, I-20126 Milano, Italy*

²*INAF – Osservatorio Astronomico di Brera, via E. Bianchi 46, I-23807 Merate, Italy*

³*INAF – Osservatorio Astronomico di Trieste, Via Tiepolo 11, I-34131, Trieste, Italy*

2 February 2017

ABSTRACT

The properties of the radiation emitted by the accretion disk around rotating black holes have been studied only recently, due to the difficulties in calculating relativistic effects caused by the strong gravitational field and to the large velocities of the matter in the vicinity of the last stable circular orbit. A numerical approach is required, as done by Li and collaborators, for the case of star-sized black holes. In this paper we build upon this numerical treatment trying to interpolate the numerical results with simple analytical functions, that can be used for black holes of any size and spin. We especially emphasize the pattern of the produced radiation, namely the flux produced as a function of the angle θ between the line of sight and the normal of the disk, and the rotation of both the black hole and the disk. Light bending and relativistic beaming concur to enhance the flux observable at large θ (i.e. edge on), contrary to the standard $\cos \theta$ pattern of disk emission around Schwarzschild black holes.

Key words: galaxies: active — (galaxies:) quasars: general — black hole physics — accretion, accretion disks

1 INTRODUCTION

In the last two decades we learnt that at the core of almost every galaxy there is a supermassive black hole (SMBH), now inactive in most cases, but that was necessarily active in the past, to have accreted the mass it has. The problem of measuring the SMBH mass at the center of galaxies is crucial to assess the properties of the black hole - galaxy core mass correlation (e.g. Magorrian et al. 1998, Gebhardt et al. 2000, Ferrarese & Merrit 2000, Marconi & Hunt 2003, Häring & Rix 2004, Gültekin et al. 2009, Beifiori et al. 2012, Kormendy & Ho 2013, McConnell & Ma 2013, Reines & Volenteri 2015), and hence up to what degree we need a feedback mechanism to explain this relation, or if more than one process is needed (see the review by Fabian 2012 and references therein). Resembling the ladder of distances, there is now a similar ladder concerning black hole masses, starting from primary measurements, using masers within the accretion disk (see e.g. Greene et al. 2010, Kuo et al. 2011 and the review by Tarchi 2012), or indirectly measuring the width of the broad emission lines to know their velocity dispersion, and the reverberation mapping (e.g. Blandford & McKee 1982, Peterson 1993, Peterson et al. 2004) to know their distance from the black hole. If the two quantities are linked by gravity, we can use the virial argument and find the black hole mass (hereafter “virial method”, e.g. Vestergaard 2002, McLure & Jarvis 2002, Greene & Ho 2005, Vestergaard & Osmer 2009, McGill et al.

2008, Wang et al. 2009, Shen et al. 2011). Or, alternatively, we can fit the accretion disk spectrum that depends in the simplest case only from the black mass M and the accretion rate \dot{M} , to find both quantities through the two observables: the disk total luminosity and its peak frequency (hereafter “SED fitting method”; see Calderone et al. 2013). All these methods bear large uncertainties ($\gtrsim 0.4 - 0.5$ dex) that are built in the intermediate correlation used to find the virial mass and the systematics involved in the calibration which will lead to significant biases of these BH mass estimates (Peterson et al. 2004, Collin et al. 2006, Shen et al. 2008, Marconi et al. 2008, Kelly et al. 2009, Shen & Kelly 2010, Park et al. 2012), and in the model used for the SED fitting method: standard accretion disk, geometrically thin, optically thick down to the innermost radius (Shakura & Sunyaev 1973), often with no rotation and no relativistic effects included.

Given the importance of finding a reliable (or at least complementary) method to establish the mass of SMBHs and their spins, in this paper we study the properties of accretion disk that are still geometrically thin and optically thick down to their innermost circular orbit (R_{ISCO}), but considering Kerr black holes and accounting for Special and General relativistic effects. Many authors tried to include relativistic effects to describe the emission from the disk (e.g. Novikov & Thorne 1973, Page & Thorne 1974, Riffert & Herold 1995), and we will base our study on an numerical code developed by Li et al. (2005) called KERRBB and implemented on the interactive X-ray spectral-fitting program XSPEC (see Arnaud 1996 and references therein). This is specifically designed for Galactic

* E-mail: sam.campitiello@gmail.com

binaries, but we will show a straightforward method for extending its capabilities for black holes of any mass.

In this paper, we concentrate on a specific aspect of the problem: how to find accurate analytical formulae that can readily describe the emission pattern of the radiation emitted by disks around Kerr holes (including the case of zero spin) accounting for relativistic effects that are found by the numerical treatment, and how to pass from star-size black holes to SMBHs.

In this work, we adopt a flat cosmology with $H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_M = 0.3$, as found by Planck Collaboration XIII (2015).

2 KERRBB SOFTWARE

2.1 Basic equations

The **KERR Black Body** (KERRBB) accretion disk model is a numerical code that describes the emission from a thin, steady state, general relativistic accretion disk around a rotating black hole, using the ray-tracing technique. It is described and compared with other models in Li et al. (2005, hereafter Li05) and represents an extension of a previous relativistic model called GRAD (Hanawa 1989, Ebisawa et al. 1991), which assumes a non rotating black hole. KERRBB is the best public code for fitting accretion disk spectra because it takes into account all the relativistic effects that other models do not or do only in part. It also takes into account the effects related to the angular momentum of the black hole, that determines the inner radius of the disk (Innermost Stable Circular Orbit, i.e. ISCO), and therefore the radiative efficiency. The accretion disk emission is approximated by a *diluted* black body: the specific intensity at radius R can be written as

$$I_\nu(R) = \left[\frac{T_{\text{eff}}(R)}{T_{\text{col}}} \right]^4 B_\nu(T_{\text{col}}) \quad (1)$$

where B_ν is the Planck function, $T_{\text{eff}}(R)$ is the effective temperature of the disk given by the Stefan-Boltzmann law $F(R) = \sigma_{\text{SB}} T_{\text{eff}}^4(R)$, $F(R)$ is the total disk flux depending on the distance R from the center¹, and T_{col} is the color temperature. The latter could be higher than the effective local temperature if, for instance, a hot corona is located above and below an otherwise standard accretion disk. In this case the Comptonization process occurring in the corona can be mimicked by assuming $T_{\text{col}} > T_{\text{eff}}$.

It is true that in an AGN the hot corona is present and active, but only in the inner regions of the accretion disk, while it is unimportant at larger radii. In this case the corresponding X-ray emission cannot be approximated by a larger temperature of the disk but must be treated as a separate process. Therefore, in this work, we assumed the hardening factor $f_{\text{col}} = T_{\text{col}}/T_{\text{eff}} = 1$, and did not account for the Comptonization process.

An important feature of KERRBB is the *self-irradiation* of the disk (i.e. *returning radiation* on the disk). The outgoing flux F_{out} is the sum of the standard emitted flux F_0 calculated ignoring returning radiation, and the contribution of the ingoing component F_{in} , due to the radiation focused back onto the disk by the black hole gravity (for other details, see Section 3 and Appendix D in Li05). The *observed* specific flux $F_{\nu, \text{obs}}$, can be expressed as (with

$G = c = h = 1$):

$$F_{\nu, \text{obs}} = 2g^3 f_{\text{col}}^{-4} E_{\text{em}}^3 \int_{\Omega} \frac{d\Omega_{\text{obs}}}{\exp[E_{\text{em}}/k_B T_{\text{col}}(R)] - 1} \quad (2)$$

where g is the redshift of the photon and E_{em} the photon energy when it is emitted from the disk. If we account for limb darkening, then we should add the extra term $Y(\theta)$ (see §2.2 and Eq. (4)). The color temperature T_{col} is defined as:

$$T_{\text{col}}(R) = f_{\text{col}} T_{\text{eff}}(R) = f_{\text{col}} \left[\frac{F_{\text{out}}(R)}{\sigma_{\text{SB}}} \right]^{1/4} \quad (3)$$

2.2 Relativistic effects

The effects of Special and General relativity caused by the black hole gravitational field, strongly modify the emission pattern of the accretion disk. These effects are more important in the inner region of the disk (closer to the black hole) than in the outer region (in KERRBB the accretion disk extends from the inner most stable orbit R_{ISCO} to $10^6 R_g$, Li05). The main relativistic effects are:

- *Doppler beaming* — Matter orbiting close to the black hole moves at relativistic velocities. This causes *relativistic aberration*, *time dilation* and *blue/red-shifting* (see below).
- *Gravitational redshift* — Electromagnetic radiation, produced in a deep potential well is observed redshifted by a distant observer². This effect becomes more important as gravity increases, i.e. closer to the black hole
- *Self irradiation of the disk* — Photons emitted by the inner disk can irradiate the outer regions of the disk itself or can be captured by the black hole because of the strong gravitational pull that bends their trajectories (for this reason, it is called *returning radiation*). The effect is stronger for rapidly rotating black holes, because R_{ISCO} becomes smaller for larger spins. For other details, see Fig. 1 in Li05.
- *Limb darkening* — This optical phenomenon makes the central part of the disk brighter than its edge. For an electron-scattering atmosphere, it can be described by a function $Y(\theta)$ of the angle between the wave vector of the emitted photons and the normal of the disk surface: $Y(\theta) = 1$ for isotropic emission. $Y(\theta) = \frac{1}{2} + \frac{3}{4} \cos \theta$ for limb-darkened emission (Chandrasekhar 1950, Cunningham 1975, Li et al. 2005). Hence, the outgoing intensity is related to the outgoing flux by

$$I_{\text{out}} = \frac{1}{\pi} F_{\text{out}} Y(\theta) \quad (4)$$

This effect modifies Eq. (2) with the extra term $Y(\theta)$ in the integral.

- *Frame dragging* — (also known as Lense-Thirring effect). This process is related to a rotating black hole that drags the space-time itself: during its fall towards the black hole, a particle will gain angular momentum that forces it to a co-rotation with the black hole spin, even in the case of a radial free fall. When $a = 0$ (Schwarzschild black hole), such effect is null.

- *Inner torque* — In the standard theory of accretion disks, the torque at the inner edge of the disk is assumed zero. This is valid for non-magnetized or weakly magnetized flows and for thin disks (Muchotrzeb & Paczynski 1985, Abramowicz & Kato 1989, Afshordi & Paczynski 2003). It has been suggested that a non-zero

¹ The general form of $F(R)$, depending also on the black hole spin, is described in Page & Thorne 1974.

² About this effect, Li05 corrected the wrong formula in the GRAD code for the redshift factor.

torque can arise from the connection between the disk and the central black hole by a magnetic field (Krolik 1999, Gammie 1999, Li 2000, Wang et al. 2002), but the problem is still under debate. A torque $g \geq 0$ at the inner boundary of the accretion disk modifies the total power of the disk as follows (with $G = c = h = 1$):

$$L_{\text{disk}} = g_{\text{in}} \Omega_{\text{in}} + \eta \dot{M} \quad (5)$$

where Ω_{in} is the angular velocity of the inner boundary, η is the radiative efficiency, \dot{M} is the black hole accretion rate. By defining the dimensionless parameter $\zeta = g_{\text{in}} \Omega_{\text{in}} / (\eta \dot{M})$, Eq. (5) becomes

$$L_{\text{disk}} = (1 + \zeta) \eta \dot{M} = \eta \dot{M}_{\text{eff}} \quad (6)$$

where $\dot{M}_{\text{eff}} = \dot{M}(1 + \zeta)$ is the *effective* accretion rate. When $\zeta = 0$, the solution corresponds to a standard Keplerian disk with zero torque at the inner boundary. For other details, see Fig. 7 and Fig. 8 in Li05.

- *Light-bending* — Photon trajectories are bent because of the strong gravitational pull exerted by the central black hole. Since R_{ISCO} decreases for larger spins, this effect becomes more important for larger spins. As a result of light-bending, the photons initially emitted in a direction normal to the disk, are bent to larger angles, increasing the radiation observed edge on (see Fig. 1). For other details, see Fig. 4 and Fig. 9 in Li05.

The effect of light-bending on the photon emission pattern from the accretion disk is sketched in Fig. 1. The top panel shows a Shakura-Sunyaev accretion disk (Shakura & Sunyaev 1973). In this model all relativistic effects are neglected and even the photons emitted close to $R_{\text{ISCO}} = 3R_S$ are not bent by gravity. The KERRBB code instead accounts for all relativistic effects even if the disk does not rotate. This case is sketched in the central panel. The dotted blue circle around the black hole contains the region where relativistic effects are stronger. The photons produced by the outer regions are not affected by the gravitational field of the black hole and are not (or only moderately) bent: for this reason, the Shakura-Sunyaev and KERRBB models are similar in the low frequency part, because the flux at these frequencies is produced by regions far from the black hole where relativistic effects are weak and can be neglected. Instead, photons emitted inside the strong influence region (i.e. inside the blue circle) are strongly affected by light-bending: these energetic photons are *lost* in one direction and *appear* in another. Furthermore, some photons emitted close to the black hole have trajectories so bent that could be directed towards the disk itself: these energetic photons can be identified as the returning radiation onto the disk that is re-emitted as an additional component of the disk flux. The bottom panel shows a KERRBB model with $a/M = 0.998$: this is the case of the strongest light-bending. In fact the extreme spin moves the inner radius of the disk down to almost the event horizon: correspondingly, the radiative efficiency is maximum, due to the enhanced emission of high energy radiation (see Fig. (8)). All these features make the disk emission brighter than the previous cases.

The sketch can be useful also for large viewing angles: in this case photons emitted in the outer region can intercept the strong influence region (blue circle) and can be bent in other directions. However the main effect is that energetic photons emitted in the inner region of the disk can be seen by observers at large angles: this explains why the KERRBB model with a large spin has an observed bolometric luminosity almost constant while varying the viewing angle (see Fig.(8)).

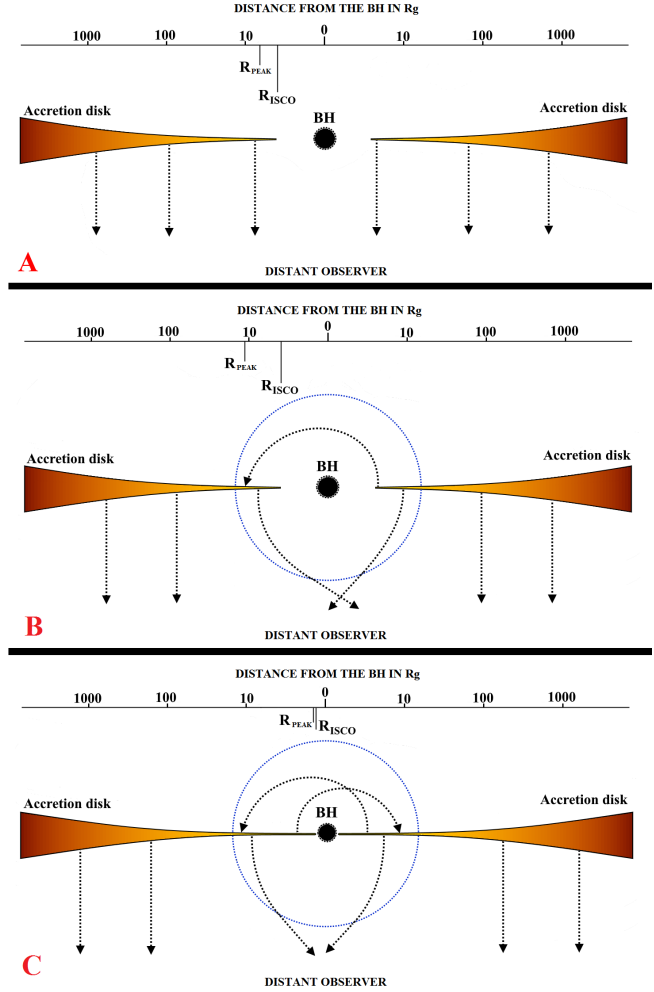


Figure 1. Schematic view of the photon emission pattern of an accretion disk around a black hole. A distant observer is at the bottom of the figures and sees the disk face-on. *Panel A:* Shakura-Sunyaev case. The absence of relativistic effects (most notably, light-bending) makes all the photons emitted from the disk go straight towards a distant observer with no modification. Also there is no returning radiation onto the disk because the light-bending effect is absent. *Panel B:* $a/M = 0$ KERRBB case. The circular region around the BH is where the effects, like light-bending, are more important. Photons emitted from the outer part of the disk go straight to the distant observer because the gravitational field of the black hole is not so strong to bend their trajectories. Photons initially emitted in the direction normal to the disk (in its inner regions), cannot reach the distant observer anymore because their trajectories are bent in different directions by the strong gravitational field. Together with the gravitational redshift (neglected in the Shakura-Sunyaev model), this explains why the KERRBB spectrum (with $a/M = 0$) is less bright than the Shakura-Sunyaev one when viewed face-on. Furthermore, some closest photons to the black hole have trajectories so bent that could be directed towards the disk itself: these energetic photons can be identified as the returning radiation onto the disk that is re-emitted as an additional component of the disk flux. *Panel C:* $a/M = 0.998M$ KERRBB case. Photons emitted in the outer region go straight to the distant observer. Inner photons have trajectories bent by the strong gravitational field. The most energetic photons could be strongly bent in all directions and some of them can be directed onto the disk. The strong relativistic effects and the high radiative efficiency make the emission brighter than previous cases, even if the disk is edge-on. The top axis is the logarithmic distance from the central black hole in unit of R_g : R_{ISCO} and radius R_{PEAK} (radius at which the flux is maximized) are shown as well.

2.3 Main parameters

The KERRBB code is based on 4 main parameters, listed below:

(i) *Black hole mass* M : in unit of solar masses M_\odot . KERRBB is designed for stellar black holes hence the black hole mass range is from 0.01 to 100 solar masses. Fig. 2 shows how accretion disk spectra scale with different black hole masses. This will be useful to scale the KERRBB results from a stellar to a supermassive black hole (see §3).

(ii) *Accretion rate* \dot{M} : in unit of 10^{18} g s^{-1} . The range, designed for stellar black hole, is from 0.01 to 1000. This parameter corresponds to the *effective* accretion rate described in Eq. (6) and therefore depends on the inner torque ζ . In this work, we assume that the inner torque is always null, so the accretion rate is simply the one of the disk $\dot{M} = \dot{M}_{\text{eff}}$. Fig. 2 and Fig.3 show how accretion disk spectra scale with different accretion rates.

(iii) *Black hole spin* a : in unit of M , the ratio a/M is dimensionless. KERRBB allows to compute the spectrum for all spins between the values $a/M = -1$ and $+0.9999$. In this work, we have considered this range, even if it goes beyond the canonical maximum spin $a/M = 0.9982$ (Thorne 1974). Fig. 3 shows how the accretion disk spectrum changes with the spin.

(iv) *Viewing angle of the disk* θ : between the normal to the disk and the line of sight. KERRBB allows to compute the spectrum in the range $0^\circ \leq \theta \leq 85^\circ$. In the case of a classical Shakura-Sunyaev model, the flux (and so the luminosity) scales as $\propto \cos \theta$ (Shakura & Sunyaev 1973, Calderone et al. 2013). In the KERRBB model the accretion disk luminosity does not follow this simple scaling because of the relativistic effects. The viewing angle at which the observer sees the maximum luminosity is larger than $\theta = 0^\circ$ and depends on the black hole spin. See Table 2.

Along with these main parameters, KERRBB allows to regulate the influence of the other features described above. For our work, we considered the self-irradiation of the disk, in order to have a more realistic description of the disk emission. On the other hand, we neglected the effects of limb-darkening and inner torque ζ . Regarding the hardening factor f_{col} , we chose to set it equal to 1, because the Comptonization effect is over-simplified if treated with this approximation, as already discussed in §2.1.

3 SCALING WITH BLACK HOLE MASS AND ACCRETION RATE

As shown in Fig. 2 and Fig. 3, the changes in accretion rate \dot{M} and black hole mass M shift the spectrum peak by a precise value related to the two parameters themselves. For the classical description of a geometrically thin and optically thick accretion disk around a non rotating black hole, given by the Shakura-Sunyaev model, the position of the peak frequency ν_p and the peak luminosity $\nu_p L_{\nu_p}$ scale with the mass and the accretion rate according to Calderone et al. (2013). These equations, adapted for the *observed* disk emission³, can be written as:

$$\frac{\nu_p}{[\text{Hz}]} = \mathcal{A} \left(\frac{\eta}{0.1} \right)^{3/4} \left[\frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right]^{1/4} \left[\frac{M}{10^9 M_\odot} \right]^{-1/2} \quad (7)$$

$$\frac{\nu_p L_{\nu_p}}{[\text{erg/s}]} = \mathcal{B} \left(\frac{\eta}{0.1} \right) \left[\frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right] \cos \theta \quad (8)$$

³ In Calderone et al. 2013, these equations are related to the *total* disk emission.

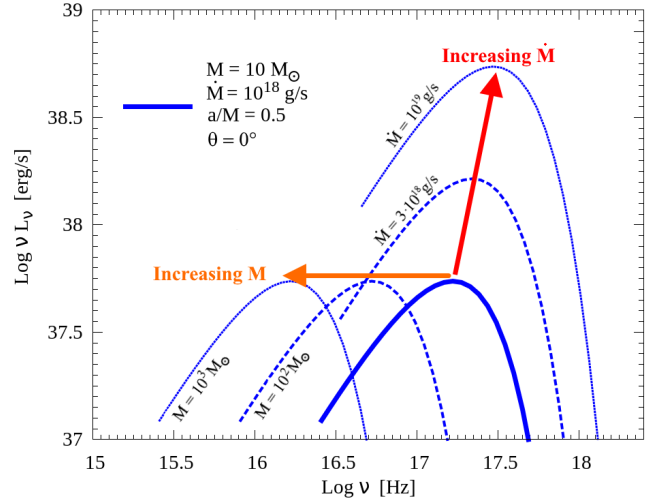


Figure 2. KERRBB accretion disk spectra with different masses and accretion rates. The initial $10M_\odot$ spectrum (blue solid line) moves horizontally (i.e. in frequency) when the black hole mass changes (orange arrow), diagonally (i.e. in both frequency and luminosity) when the accretion rate changes (red arrow). The position of the peak in frequency ν and luminosity νL_ν , for a given spin value, follows the same shifting equations found by Calderone et al. (2013): corrected Eq. (9) and Eq. (10) (depending also on the black hole spin a) can be found using KERRBB results (in the specific case of a face-on disk).

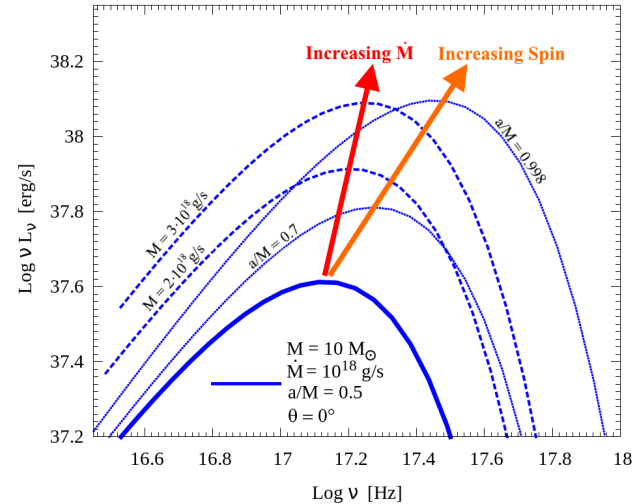


Figure 3. KERRBB accretion disk spectra with different accretion rates and spins, but same black hole mass ($10M_\odot$). When \dot{M} increases, the spectrum peak increases also (red arrow), but less than the increase related to the increase of the spin (orange arrow). The low frequency part of the spectrum, when a changes, remains constant, but this occurs at frequencies not visible in the figure. The low frequency flux is produced by the outer regions of the disk, unaffected by the black hole spin. About this effect, see also Fig. (8).

where η is the radiative efficiency and the constants are $\text{Log } \mathcal{A} = 15.25$ and $\text{Log } \mathcal{B} = 45.66$. Note that for the Shakura-Sunyaev model, $\eta \sim 0.1$ so the $\eta/0.1 \sim 1$ in both equations.

The two equations, for a fixed viewing angle θ , give univocally the position of the spectrum peak corresponding to a given mass and accretion rate for a Shakura-Sunyaev model. For KERRBB, Eq. (7) and Eq. (8) have the same form, but depend also on the spin, that changes the efficiency η and the radiation pattern, no longer

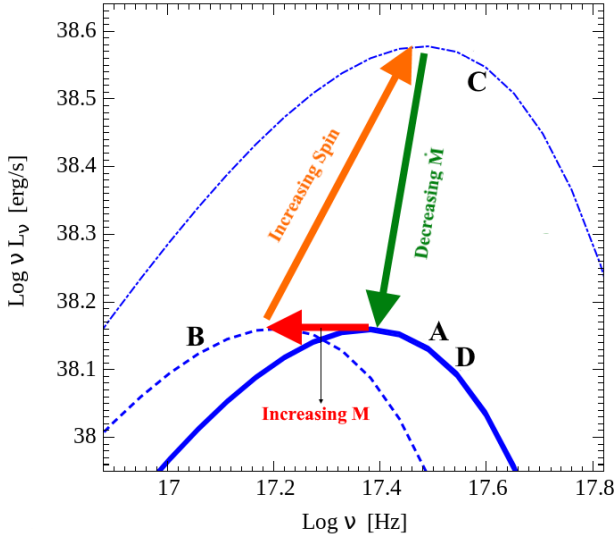


Figure 4. We can obtain the same KERRBB spectrum (i.e. the emission peak in the same position) for different values of the black hole mass M , accretion rate \dot{M} and spin a/M . There is therefore a *family* of solutions. Consider the spectrum A (thick solid blue line), and suppose to increase M at constant \dot{M} and spin. We would obtain the spectrum B. Now suppose to increase the spin, but keep M and \dot{M} constant. Since a larger spin corresponds to a larger efficiency, we obtain the spectrum C. Finally, suppose to *decrease* \dot{M} : we can obtain the spectrum D, which is almost exactly equal to the initial spectrum A (they slightly differ in the high frequency, exponential part).

equal to $\cos \theta$. The mass and accretion rate dependences are the same because the assumption of a geometrically thin and optically thick disk is valid for both models. By using KERRBB results, it is possible to find new equations, applicable for Kerr black holes. They can be written as

$$\frac{\nu_p}{[\text{Hz}]} = \mathcal{A} \left[\frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right]^{1/4} \left[\frac{M}{10^9 M_\odot} \right]^{-1/2} g_1(a, \theta) \quad (9)$$

$$\frac{\nu_p L_{\nu_p}}{[\text{erg/s}]} = \mathcal{B} \left[\frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right] g_2(a, \theta) \quad (10)$$

The functions g_1 and g_2 describe the dependencies on the black hole spin a (and hence also on the radiative efficiency) and the viewing angle θ . We derived a functional form for both $g_1(a, \theta = 0^\circ)$ and $g_2(a, \theta = 0^\circ)$ by fitting the variation of the spectrum peak position at different spin values and fixed $\theta = 0^\circ$:

$$\begin{aligned} g_i(a, \theta = 0^\circ) &= \alpha + \beta x_1 + \gamma x_2 + \delta x_3 + \epsilon x_4 + \zeta x_5 + \iota x_6 \\ x_n &\equiv \log(n - a) \quad i = 1, 2 \end{aligned} \quad (11)$$

where a is the adimensional black hole spin. Table 1 gives the parameter values of the two functions. The accuracy of Eq. 11 is of the order of $\sim 1\%$. It is important to note that the peak frequency and luminosity, described by Eq. (9) and Eq. (10), are degenerate in mass, accretion rate and spin (if θ is fixed). In other words, the same spectrum can be fitted by a family of solutions, by changing the black hole mass, accretion rate and spin in a proper way. Fig. 4 shows this degeneracy: starting from model A, suppose to increase M at constant \dot{M} and spin: we would obtain the spectrum B. Now suppose to increase the spin, but keep M and \dot{M} constant: since a larger spin corresponds to a larger efficiency, we obtain the spectrum C. Finally, suppose to *decrease* \dot{M} : we can obtain the spec-

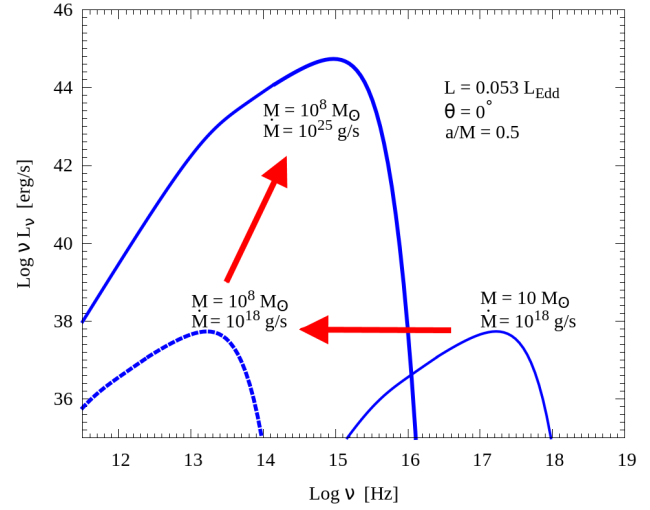


Figure 5. Scaling mass and accretion rate from a stellar black hole to a supermassive black hole. The initial model (thin solid blue line) is first shifted in frequency by changing the mass value then in luminosity (and frequency) by changing the accretion disk value according to Eq. (12) and Eq. (13). In this way, it is possible to describe the accretion disk emission data related to supermassive black holes with KERRBB. Note that the initial and the final spectra have the same Eddington ratio $L/L_{\text{Edd}} = 0.053$. In the Appendix, Eq. (12) and Eq. (13) are expressed as a function only of the black hole mass, by considering that the Eddington ratio of the initial and the final models must be equal.

trum D, which is equal to the initial spectrum A. The overlapping of two models can be done because, as shown in Fig. 3, the accretion rate and the spin move the spectrum peak in different directions. Strictly speaking, model A and D are not exactly equal: there are very small differences in the high frequency, exponential part of the spectra: for larger spin values, the exponential tail is brighter.

Since KERRBB spectrum moves on the $\text{Log} \nu, \text{Log} \nu L_\nu$ plane according to the proportionalities of Eq. (9) and Eq. (10), we can scale the spectrum from stellar black hole mass to a supermassive black hole mass, with the same spin value. It is important to note that this procedure can be done if the emission processes from an accretion disk around a star-sized and a supermassive black hole are the same. In our case, in both systems the emission is required to be a multi-color blackbody. Some authors have pointed out some problems with the multi-color blackbody interpretation for SMBHs (Koratkar & Blaes 1999, Davis & Laor 2011), but here we assume that it is a good enough approximation in order to use Eq. (9) and Eq. (10). Fig. 5 shows an example of our procedure: starting from a stellar black hole mass (thin solid blue line), the spectrum can be shifted in frequency and luminosity by quantities related to the initial and the final mass:

$$\frac{\nu_{\text{fin}}}{\nu_{\text{in}}} = \left[\frac{\dot{M}_{\text{fin}}}{\dot{M}_{\text{in}}} \right]^{1/4} \sqrt{\frac{M_{\text{fin}}}{M_{\text{in}}}} \quad (12)$$

$$\frac{\nu L_{\nu_{\text{fin}}}}{\nu L_{\nu_{\text{in}}}} = \frac{\dot{M}_{\text{fin}}}{\dot{M}_{\text{in}}} \quad (13)$$

In this way, it is possible to describe the accretion disk emission data related to supermassive black holes with KERRBB. In the Appendix, Eq. (12) and Eq. (13) are expressed as a function only of the black hole mass, by requiring the Eddington ratio of the initial and the final models equal [Eq. (A4) and Eq. (A5)].

Parameters	α	β	γ	δ	ϵ	ζ	ι
g_1	1001.3894	-0.061735	-381.64942	8282.077258	-40453.436	66860.08872	-34974.1536
g_2	2003.6451	-0.166612	-737.3402	16310.0596	-80127.1436	132803.23932	-69584.31533

Table 1. Parameters of the functions g_1 and g_2 in Eq. (9) and Eq. (10), written as a general functional (11), in the case with the viewing angle 0° . Using KERRBB results, it is possible to find similar equations for the cases with the disk viewing angle $> 0^\circ$.

4 EMISSION PATTERN

4.1 Accretion disk bolometric luminosity

In the classical non relativistic Shakura-Sunyaev case, the bolometric luminosity *observed* at an angle θ is related to the *total* disk luminosity L_d by the following expression (Calderone et al. 2013):

$$L_{SS}^{obs}(\theta) = 2 \cos \theta L_d = 2 \cos \theta \eta \dot{M} c^2 \quad (14)$$

As mentioned before, this is no longer valid in the general relativistic case for a Kerr black hole. In this case, the emission pattern depends not only on \dot{M} and θ but also on the black hole spin a . Our task was to find an analytic expression for it. To this aim, we wrote the observed bolometric luminosity as:

$$L_{Kerr}^{obs} = f(\theta, a) \eta(a) \dot{M} c^2 \quad (15)$$

where the radiative efficiency $\eta(a)$ depends on the black hole spin. We required that this last equation satisfies:

$$\frac{1}{4\pi} \int_{\Omega} L_{Kerr}^{obs}(\theta, a) d\Omega = \eta(a) \dot{M} c^2 \quad (16)$$

Using Eq. 15, this implies:

$$\int_0^{\pi/2} f(\theta, a) \sin \theta d\theta = 1 \quad (17)$$

First, we considered the bolometric luminosity of the spectra given by the KERRBB code, fixing the values \dot{M} , M , a and varying only the viewing angle. In this way, we derived the emission pattern numerically. Therefore we looked for an analytic expression that could interpolate the numerical results with a good accuracy. Since we wanted to compare the final pattern with the simple $2 \cos \theta$ term in Eq. (14), we assumed that the first term of the final analytic expression (function of θ) is just $\cos \theta$. Therefore, considering Eq. (15), we assumed:

$$L_{Kerr}^{obs} = \underbrace{[\cos \theta \cdot (\text{terms functions of } \theta)]}_{=f(\theta, a)} \eta(a) \dot{M} c^2 \quad (18)$$

We obtained a good match using the following functional form:

$$L_{Kerr}^{obs} = \underbrace{A \cos \theta [1 - (\sin \theta)^C]^B [1 - E(\sin \theta)^F]^D}_{=f(\theta, a)} \eta(a) \dot{M} c^2 \quad (19)$$

The accuracy of Eq. (19) is of $< 1\%$, hence represents a very good approximation. We also tried a polynomial function with the same number of parameter but the accuracy was lower ($> 1\%$) at small and large angles, for different spin values. All the parameters A, B, C, D, E and F are functions of the spin a . We then repeated this analysis using different spin values. Table B1 in Appendix lists the values of the parameters for different spins. With this approach, we studied how the values of A, B, C, D, E and F change by changing the spin. Again, we looked for an analytical function that interpolates the numerical results (see Fig. B1 and Fig. B2 in the Appendix). We found that a good representation is given by:

$$\begin{aligned} \mathcal{H}(a) &= \alpha + \beta x_1 + \gamma x_1^2 + \delta x_1^3 + \epsilon x_1^4 + \iota x_1^5 + \kappa x_1^6 \\ x_1 &= \log(1 - a) \end{aligned} \quad (20)$$

Spin value [a/M]	θ_{max}	Spin value [a/M]	θ_{max}
0.998	63.9°	0.6	0.3°
0.98	46.4°	0.5	0.1°
0.95	24.6°	0.4	0.1°
0.94	19.1°	0.3	0.028°
0.92	12.5°	0.2	0.009°
0.9	8.7°	0.1	0.002°
0.8	2.2°	0	0.0005°
0.7	0.8°	-1	0°

Table 2. Values of the viewing angle θ_{max} at which the disk bolometric luminosity is maximized, for different spins. Note the extreme effect at spin $a/M = 0.998$ and little fluctuations from 0° for smaller spins. In a sense, these results show the deviation from the $\cos \theta$ -law at different spins.

The values of $\alpha, \beta, \gamma, \delta, \epsilon, \iota, \kappa$ of Eq. 20 are in Table B2 in the Appendix. The accuracy of Eq. 20 is of $\sim 1\%$. As an example, in the case with $a/M = 0$ ($\eta = 0.057$), $\dot{M} = 10^{18}$ g/s, the function that interpolates the observed disk luminosity for different angles θ is:

$$\frac{L_{Kerr}^{obs}(\theta)}{[\text{erg/s}]} = 8.475 \cdot 10^{37} \frac{\cos \theta [1 - 0.958 \cdot (\sin \theta)^{6.675}]^{0.121}}{[1 - (\sin \theta)^{1.918}]^{0.202}} \quad (21)$$

Following the parallel structure of emission patterns in KERRBB and Shakura-Sunayev, we did not expect any dependence of the function $f(\theta, a)$ on the black hole mass M and indeed we did not find any. Therefore, the bolometric luminosity L_{Kerr}^{obs} is only a function of θ, a and \dot{M} , as written in Eq. (15).

4.2 Pattern

Through Eq. (15), whose explicit form is Eq. (19), it is possible to study the emission pattern of the Kerr black hole accretion disk. For the Shakura-Sunayev model, the pattern follows the $\cos \theta$ -law [Eq. (14)], the maximum luminosity is at 0° and it is null at 90° . KERRBB radically changes the θ -dependence and the angle at which the observed luminosity is maximized (θ_{max}) is not necessarily 0° . It is possible to find this θ_{max} by putting equal to 0 the derivative of Eq. (19) with respect the viewing angle θ . Setting $\eta(a) \dot{M} c^2 = 1$, we obtain:

$$\begin{aligned} \frac{\partial L_{Kerr}^{obs}(\theta, a)}{\partial \theta} &= [1 - (\sin \theta)^C] \cdot [1 - E(\sin \theta)^F] + \\ &+ CB(\sin \theta)^{C-2} \cos^2 \theta [1 - E(\sin \theta)^F] + \\ &+ FED(\sin \theta)^{F-2} \cos^2 \theta [1 - (\sin \theta)^C] = 0 \end{aligned} \quad (22)$$

The angles θ_{max} increases for increasing spin a . As an example, $\theta_{max} \simeq 64^\circ$ for $a/M = 0.998$ and $\simeq 25^\circ$ for $a/M = 0.95$. By reducing the spin value, θ_{max} approaches 0° (see Table 2 for other spin values). Note that Eq. (22) is equal to zero also for

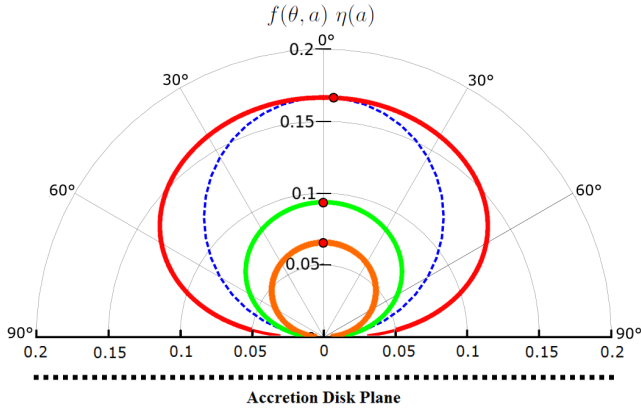


Figure 6. Emission pattern for different models: the radial axis is the *observed efficiency*, given by $\eta_{\text{obs}} \equiv f(\theta, a)\eta(a)$. The classical non relativistic Shakura-Sunyaev model (dashed blue line) is compared with $a/M = -1$ (orange line), $a/M = 0$ (green line) and $a/M = 0.797$ (red line) KERRBB patterns. Note that for the Shakura-Sunyaev model, the observed efficiency is $\eta_{\text{obs}} = 2 \cos \theta \eta_{\text{SS}} \approx 0.17 \cos \theta$. The $a/M = 0.797$ case describes the most similar KERRBB model to a Shakura-Sunyaev with same parameters (mass and accretion rate) at $\theta = 0^\circ$. At different viewing angles, the KERRBB model is brighter. Note also, how for the Shakura-Sunyaev model, the emission has a circular pattern while for KERRBB, the pattern is “warped” by the relativistic effects. Red dots indicate the angle at which the KERRBB luminosity is maximized (Table 2 and Fig. 7).

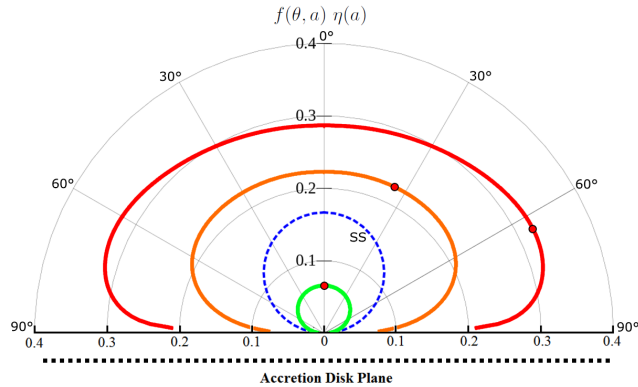


Figure 7. Emission pattern for different models: the radial axis is the *observed efficiency*, given by $\eta_{\text{obs}} = f(\theta, a)\eta(a)$. The classical non relativistic Shakura-Sunyaev model (dashed blue line) is compared with $a/M = -1$ (green line), $a/M = 0.95$ (orange line) and $a/M = 0.998$ (red line) KERRBB patterns. Note how, for the extreme KERRBB, the emission is strongly modified with respect to the others. The angles at which the KERRBB luminosity is maximized are indicated with red dots: for $a/M = 0.998$, $\theta_{\text{max}} \simeq 64^\circ$, and for $a/M = 0.95$, $\theta_{\text{max}} \simeq 25^\circ$ (see Table 2 for other spin values).

$\theta = 90^\circ$: this solution must not be considered because Eq. (19) is defined in the interval $[0^\circ : 85^\circ]$ (see §2.3).

In order to plot the different emission patterns for different angles and spins, we kept constant the value of the accretion rate \dot{M} in Eq. (15) and computed the observed efficiency defined by $\eta_{\text{obs}} \equiv f(\theta, a)\eta(a)$. Fig. 6 shows the emission pattern for different models: the radial axis is the observed efficiency plotted at different viewing angles. For the classical non relativistic Shakura-Sunyaev model (dashed blue line), $\eta_{\text{obs}} = 2 \cos \theta \eta_{\text{SS}} \approx 0.17 \cos \theta$. It is compared with the $a/M = -1$ (orange line), 0 (green line)

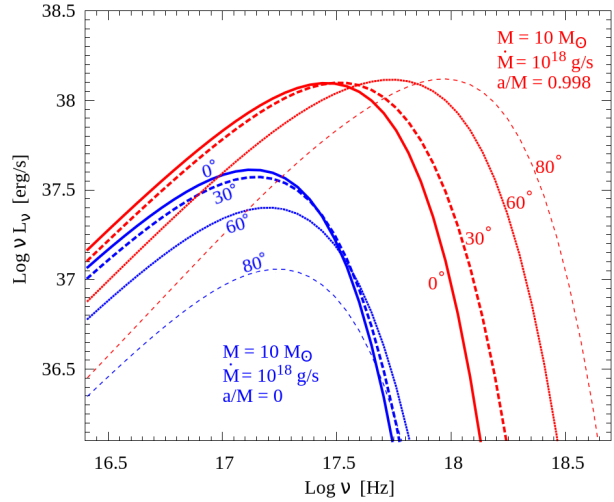


Figure 8. KERRBB model behavior at different angles and spins. All models have the same M and \dot{M} . When $a/M = 0$ case (blue lines), the disk luminosity decreases increasing the viewing angle in a way similar to $\cos \theta$. When the spin is maximal $a/M = 0.998$ (red lines), light-bending effect, along with Doppler beaming and gravitational redshift are so strong that the luminosity is almost the same, even at large angles. In this case, the trajectories of the energetic photons, produced close to the black hole horizon, are bent in all directions making the radiation intensity to be almost the same at all viewing angles.

and 0.797 (red line) KERRBB patterns. For Shakura-Sunyaev, the emission is given by a circle: for KERRBB, this circle is “warped” by relativistic effects. The $a/M = 0.797$ case describes the most similar KERRBB model to a Shakura-Sunyaev, at $\theta = 0^\circ$, with the same black hole mass and accretion rate⁴: in order to emit the same luminosity, the KERRBB model must have a larger efficiency, hence a larger spin value. In this way, the two models are equivalent around $\theta = 0^\circ$ but at different viewing angles, the KERRBB model is brighter because of the strong relativistic effects. Fig. 7 is equivalent to Fig. 6: the Shakura-Sunyaev is compared with the $a/M = -1$ (green line), 0.95 (orange line) and 0.998 (red line) KERRBB patterns. Note how, for the extreme KERRBB, the emission is strongly modified with respect to the other cases. Red dots indicate the angles at which the emission pattern is maximized.

The strong modification of the spectrum emission due to the combination of viewing angle and spin is also visualized in Fig. 8: the blue spectra correspond to $a/M = 0$, the red ones are the same with $a/M = 0.998$ (M and \dot{M} are constant). In the $a/M = 0$ case, relativistic effects are very weak and the spectrum seem to follow the classical $\cos \theta$ - law. Instead, in the $a/M = 0.998$ cases, the emitted luminosity is almost constant, even for the largest viewing angles. This is due to the combination of different relativistic effects (Doppler beaming, gravitational redshift and light-bending) along with the black hole spin: the trajectories of the energetic photons coming from the innermost region of the disk (that are very close to the horizon for $a/M \rightarrow 1$), are bent in all directions and the intensity of radiation is almost the same at all viewing angles.

⁴ For the same mass (or accretion rate) value, the precise overlapping has a smaller spin (~ 0.76) but a different accretion rate (or mass) value (differences of the order of $\sim 1\%$). The overlapping case with $a/M = 0.797$ is the closest one with the same M and \dot{M} , even if imprecise.

5 DISCUSSION AND CONCLUSIONS

In this work, we studied the radiation and emission pattern from an accretion disk around a spinning black hole. Our aim was to build an analytic approximation of the emission pattern, given by the numerical model KERRBB (Li et al. 2005), already developed for X-ray binaries and accounting for all the relativistic effects (i.e. Doppler beaming, gravitational redshift, self-irradiation of the disk, light-bending), in order to efficiently extend its use to supermassive black holes.

We first tested some features of the model comparing it to the classical non relativistic description by Shakura & Sunyaev (1973): we verified how the relativistic effects affect the emission profile of the disk, mainly in the vicinity of the black hole. Afterwards, we focused on the changes in the spectrum implied by changes in the 4 main parameters of the model: *black hole mass*, *accretion rate*, *spin* and *inclination angle of the disk* with respect to the line of sight. A first analysis showed that the KERRBB model is affected by changes in mass and accretion rate in the same way as a standard Shakura-Sunyaev model: the assumption of a geometrically thin and optically thick disk is valid for both models hence the disk emission scales with the same black hole mass and accretion rate proportionalities found by Calderone et al. (2013). This useful information allows to shift the KERRBB spectrum, from a star-sized to a supermassive black hole, by using Eq. 12 and Eq. 13 (or equivalently Eq. A4 and Eq. A5 if, in the transformation, the Eddington ratio of the initial and the final model is kept constant). Also, using KERRBB results, the equations related to the peak frequency and luminosity of the spectrum, found by Calderone et al., can be adapted to rotating black holes: new equations 9 and 10 and Fig. 4 showed that the same spectrum can be reproduced by a family of solutions, with different value for the BH mass, the accretion rate and the spin.

The most important part of this work is related to the analysis of the emission pattern, i.e. how the observed luminosity L^{obs} of the accretion disk depends on the viewing angle and the BH spin. The presence of relativistic effects makes the KERRBB $L_{\text{Kerr}}^{\text{obs}}$ different from the classical Shakura-Sunyaev $L_{\text{SS}}^{\text{obs}}$: this latter can be analytically found simply by considering anisotropic the emitted flux of the accretion disk, leading to Eq. 14 (Calderone et al. 2013). By using KERRBB results, we tried to recover an analytic expression depending only on the viewing angle and the black hole spin, and we obtained a phenomenological function $f(\theta, a)$ that well approximates the variation of the observed luminosity at different θ and a values: the observed luminosity can be written as in Eq. 19.

The availability of an analytic expression for the bolometric luminosity related to a spinning black hole can be extremely useful in the analysis of the emission features of both stellar and supermassive active black holes. These features can be visualized in Fig. 6 and Fig. 7 (see captions for the diagram details). The analysis of the pattern showed that:

- A larger spin implies a larger bolometric luminosity. The increase of luminosity and the relativistic effects on the pattern are more pronounced for $a/M \gtrsim 0.8$.
- The KERRBB model with $a/M = 0$ is dimmer than the equivalent Shakura-Sunyaev at fixed M and \dot{M} : this is due to the presence of relativistic effects and a smaller radiative efficiency ($\eta_{\text{KERR}} = 0.057$).
- At a fixed viewing angle, there is a KERRBB model equivalent to a Shakura-Sunyaev one with the same parameters (mass and accretion rate). At $\theta = 0^\circ$, the equivalent KERRBB has spin $a/M = 0.797$. In fact, in order to emit the same luminosity, the

KERRBB emission must have a larger efficiency, hence a larger BH spin value. In this way, the two models are equivalent around $\theta = 0^\circ$ but, at larger viewing angles, the KERRBB model is brighter because of the strong relativistic effects.

- Relativistic effects modifies the pattern at different viewing angles: the simple $\cos \theta$ - law (followed by Shakura-Sunyaev) is no longer valid, mostly due to light-bending (see Fig. 1). Hence the maximum observed luminosity is no longer at $\theta = 0^\circ$ but at larger viewing values.
- The observed luminosity is maximized at different angles depending on the black hole spin: for larger spin values, θ_{max} has a larger value (see Tab. 2 for its value at different spins).

With our approach, we introduced a simple analytic tool to study the impact of different spin values on observable features of the accretion disks surrounding supermassive black holes. In upcoming publications, we will detail how our analytic approximation can be used to derive accretion and black hole features (introducing a new method to estimate the black hole spin from the disk SED fitting process), and likely be used for a wide active galactic nuclei sample.

REFERENCES

- Abramowicz, M. A. & Kato, S. 1989, ApJ, 336, 304.
 Afshordi, N. & Paczynski, B. 2003, ApJ, 592, 354.
 Arnaud, K. A. 1996, Astronomical Data Analysis Software and Systems V. ASP Conference Series, Vol. 101, eds. G. H. Jacoby and J. Barnes, p. 17.
 Beifiori, A., Courteau, S., Corsini, E. M., & Zhu, Y. 2012, MNRAS, 419, 2497.
 Blandford, R. D. & McKee, C. F. 1982, ApJ, 255, 419.
 Calderone, G., Ghisellini, G., Colpi, M., & Dotti, M. 2013, MNRAS, 431, 210.
 Chandrasekhar, S. 1950, Oxford: Oxford Univ. Press.
 Collin, S., Kawaguchi, T., Peterson, B. M., & Vestergaard, M. 2006, A & A, 456, 75.
 Cunningham, C. T. 1975, ApJ, 202, 788.
 Davis, S. W. & Laor, A. 2011, ApJ, 728, 98.
 Ebisawa, K., Mitsuda, K., & Hanawa, T. 1991, ApJ, 367, 213.
 Fabian, A. C. 2012, ARA & A, 50, 455.
 Ferrarese, L. & Merrit, D. 2000, ApJ, 539, L9.
 Gammie, C. F. 1999, ApJ, 522, L57.
 Gebhardt, K. et al. 2000, ApJ, 539, L13.
 Greene, J. E. et al. 2010, ApJ, 721, 26.
 Greene, J. E. & Ho, L. C. 2005, ApJ, 630, 122.
 Gültekin, K., Richstone, D. O., & Gebhardt, K. 2009, ApJ, 698, 198.
 Hanawa, T. 1989, ApJ, 341, 948.
 Häring, N. & Rix, H.-W. 2004, ApJL, 604, L89.
 Kelly, B. C., Vestergaard, M., & Fan, X. 2009, ApJ, 692, 1388.
 Koratkar, A. & Blaes, O. 1999 PASP, 111, 1.
 Kormendy, J. & Ho, L. C. 2013, ARA & A, 51, 51.
 Krolik, J. K. 1999, ApJ, 515, L73.
 Kuo, C. Y. et al. 2011, ApJ, 727, 2.
 Li, L.-X. 2000, ApJ, 533, L115.
 Li, L.-X., Zimmerman, E. R., Narayan, R., & McClintock, J. E. 2005, ApJ, 157, 335-370.

- Magorrian, J., Tremaine, S., Richstone, D., & R. 1998, *AJ*, 115, 228.
- Marconi, A., Axon, D. J., Maiolino, R., Nagao, T., Pastorini, G., Pietrini, P., Robinson, A., & Torricelli, G. 2008, *ApJ*, 678, 693.
- Marconi, A. & Hunt, L. K. 2003, *ApJL*, 589, L2.
- McConnell, N. J. & Ma, C.-P. 2013, *ApJ*, 764, 18.
- McGill, K. L., Woo, J., Treu, T., & Malkan, M. A. 2008, *ApJ*, 673, 703.
- McLure, R. J. & Jarvis, M. J. 2002, *MNRAS*, 337, 109.
- Muchotrzeb, B. & Paczynski, B. 1985, *Acta Astron.* 32, 1.
- Novikov, I. D. & Thorne, K. S. 1973, in *Black holes*, ed. C. De Witt and B. De Witt (New York: Gordon and Breach), 343.
- Page, D. N. & Thorne, K. S. 1974, *ApJ*, 191, 507.
- Park, D. et al. 2012, *ApJ*, 747, 30.
- Peterson, B. M. 1993, *PASP*, 105, 247.
- Peterson, B. M. et al. 2004, *ApJ*, 613, 682.
- Reines, A. E. & Volenter, M. 2015, *ApJ*, 813, 82.
- Riffert, H. & Herold, H. 1995, *ApJ*, 450, 508.
- Shakura, N. I. & Sunyaev, R. A. 1973, *AA*, 24, 337.
- Shen, J., Berk, D. E. V., Schneider, D. P., & Hall, P. B. 2008, *AJ*, 135, 928.
- Shen, Y. et al. 2011, *ApJS*, 194, 45.
- Shen, Y. & Kelly, B. C. 2010, *ApJ*, 713, 41.
- Tarchi, A. 2012, *IAU Symp.*, 287, 323 (arXiv: 1205.3623).
- Thorne, K. S. 1974, *ApJ*, 191, 507-519.
- Vestergaard, M. 2002, *ApJ*, 571, 733.
- Vestergaard, M. & Osmer, P. S. 2009, *ApJ*, 699, 800.
- Wang, D.-X., Xian, K., & Lei, W. H. 2002, *MNRAS*, 335, 655.
- Wang, J. G. et al. 2009, *ApJ*, 707, 1334.

APPENDIX A: SHIFTING EQUATIONS

Eq. (12) and Eq. (13) describe the relations that allow to rescale the initial spectrum with mass M_{in} and \dot{M}_{in} , to a new spectrum with final M_{fin} and \dot{M}_{fin} , with a fixed black hole spin. The peak frequency ν_p and luminosity $\nu_p L_{\nu_p}$ follow these relations:

$$\frac{\nu_{p,\text{fin}}}{\nu_{p,\text{in}}} = \left(\frac{\dot{M}_{\text{fin}}}{\dot{M}_{\text{in}}} \right)^{1/4} \sqrt{\frac{M_{\text{in}}}{M_{\text{fin}}}} \quad (\text{A1})$$

$$\frac{\nu L_{\nu_{p,\text{fin}}}}{\nu L_{\nu_{p,\text{in}}}} = \frac{\dot{M}_{\text{fin}}}{\dot{M}_{\text{in}}} \quad (\text{A2})$$

If, in the transformation, the Eddington ratio $\ell = L/L_{\text{Edd}}$ remains the same, the previous equations become functions only of the black hole mass. In fact, one can write:

$$L_{\text{in}} = \eta \dot{M}_{\text{in}} c^2 \quad L_{\text{fin}} = \eta \dot{M}_{\text{fin}} c^2$$

$$L_{\text{Edd},\text{in}} = C M_{\text{in}} \quad L_{\text{Edd},\text{fin}} = C M_{\text{fin}}$$

where $C = 6.9 \cdot 10^4 \text{ erg s}^{-1} \text{ g}^{-1}$. The Eddington ratios will be:

$$\ell_{\text{in}} = \frac{\eta \dot{M}_{\text{in}} c^2}{C M_{\text{in}}} \quad \ell_{\text{fin}} = \frac{\eta \dot{M}_{\text{fin}} c^2}{C M_{\text{fin}}}$$

If $\ell_{\text{in}} = \ell_{\text{fin}}$, one obtains:

$$\frac{\dot{M}_{\text{in}}}{\dot{M}_{\text{fin}}} = \frac{M_{\text{in}}}{M_{\text{fin}}} \quad (\text{A3})$$

In this way, Eq. (12) and Eq. (13) become:

$$\frac{\nu_{p,\text{fin}}}{\nu_{p,\text{in}}} = \left(\frac{M_{\text{in}}}{M_{\text{fin}}} \right)^{1/4} \quad (\text{A4})$$

$$\frac{\nu L_{\nu_{p,\text{fin}}}}{\nu L_{\nu_{p,\text{in}}}} = \frac{M_{\text{fin}}}{M_{\text{in}}} \quad (\text{A5})$$

APPENDIX B: KERRBB OBSERVED LUMINOSITY

In §4.1, we showed that an analytic expression that approximates the observed, frequency integrated luminosity of an accretion disk around a rotating black hole, can be written as:

$$\begin{aligned} L_{\text{Kerr}}^{\text{obs}} &= A \cos \theta [1 - (\sin \theta)^C]^B [1 - E(\sin \theta)^F]^D L_d \\ L_d &= \eta(a) \dot{M} c^2 \end{aligned} \quad (\text{B1})$$

where L_d is the total disk luminosity. All the parameters A, B, C, D, E, F are functions only of the black hole spin a , like the radiative efficiency $\eta(a)$. In order to find the expression for the observed luminosity, for a given value of a , it has been chosen a set of viewing angle θ (from 0° to 85°), and computed the integrals over frequency of the KERRBB spectra. We fitted these latter and found the empirical expression (B1). Parameter values for different spin values are in Tables B1.

Fig. B1 shows the bolometric luminosities of the KERRBB spectra as a function of the viewing angle, for spin $a/M = -1, -0.6, 0.6$ and 0.998 (red dots), and the fitting function (B1) (blue line). Note the different behaviors: in the cases $a/M = -1, -0.6$ and 0.6 , the bolometric luminosity, between the cases 0° and 85° , decreases by a factor of $\sim 7, 6.6$ and 4.4 , respectively⁶. In the case with $a/M = 0.998$, the strong relativistic effects make the luminosity reach a maximum value at $\sim 64^\circ$ (see Table 2), and then make it drop at larger viewing angles. The cases with 0° and 85° have almost the same luminosity, (see also Figure 8), contrary to the small-spin cases. Note how the residuals are always of the order of $\sim 0.01\%$, hence function (B1) represents a very good approximation.

Fig. B2 shows the parameters A, B, C, D, E and F for different values of the spin, in order to find their dependence on it. As shown in §4.1, the common fitting equation (blue line) can be written as:

$$\begin{aligned} \mathcal{H}(a) &= \alpha + \beta x_1 + \gamma x_1^2 + \delta x_1^3 + \epsilon x_1^4 + \iota x_1^5 + \kappa x_1^6 \\ x_1 &= \log(1 - a) \end{aligned} \quad (\text{B2})$$

The values of $\alpha, \beta, \gamma, \delta, \epsilon, \iota, \kappa$ are in Table B2. Note how the residuals are always of the order of 1% (or less), except for F (of the order of $\sim 10\%$). These large uncertainties could be reduced using more than 6 parameters for the fit, but we found it unnecessary for the aim of this work.

⁵ The KERRBB code allows to chose a viewing angle between these two limiting values.

⁶ In the case with no relativistic effects (i.e. Shakura-Sunyaev case), there would have been a factor of ~ 11.5 , between the cases 0° and 85° .

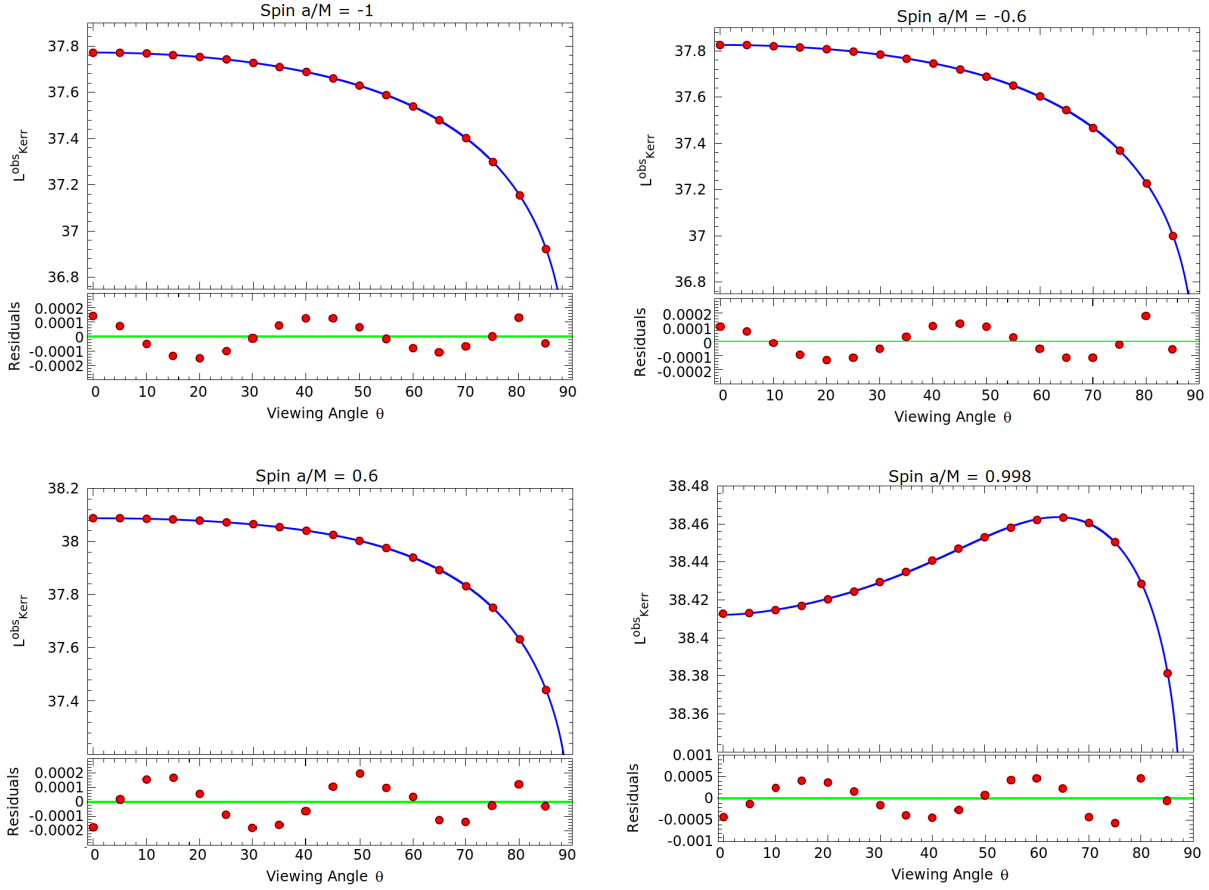


Figure B1. KERRBB disk bolometric luminosity as a function of the viewing angle of the disk, in the cases with $a/M = -1$, $a/M = -0.6$, $a/M = 0.6$ and $a/M = 0.998$. The fitting function (blue line) has the general form of (19) with different values for the parameters, in the different cases. Note the different behaviors: in the cases $a/M = -1$, -0.6 and 0.6 , the bolometric luminosity, between the cases 0° and 85° , decreases by a factor of ~ 7 , 6.6 and 4.4 , respectively. In the case with $a/M = 0.998$, the luminosity reaches a maximum value at $\sim 64^\circ$ (see Table 2) then drops at larger viewing angles.

Par	$a = 0.998M$	$a = 0.95M$	$a = 0.8M$	$a = 0.6M$	$a = 0.4M$
Log A	-0.04950 ± 0.00029	0.06055 ± 0.00029	0.13668 ± 0.00017	0.17345 ± 0.00011	0.19378 ± 0.00008
B	-0.59008 ± 0.00508	-0.46635 ± 0.00576	-0.35544 ± 0.00471	-0.28623 ± 0.00382	-0.24788 ± 0.00346
C	1.90743 ± 0.01574	1.87187 ± 0.02087	1.87208 ± 0.01959	1.87674 ± 0.01806	1.89046 ± 0.01758
D	0.16910 ± 0.00415	0.20649 ± 0.00473	0.18649 ± 0.00393	0.15944 ± 0.00311	0.14239 ± 0.00272
E	1.00344 ± 0.00121	0.99368 ± 0.00146	0.98357 ± 0.00159	0.97466 ± 0.00203	0.96833 ± 0.00251
F	12.00334 ± 0.56470	10.6638 ± 0.4250	8.75966 ± 0.26306	7.86278 ± 0.20588	7.28707 ± 0.18300
Par	$a = 0$	$a = -0.2M$	$a = -0.4M$	$a = -0.6M$	$a = -1M$
Log A	0.21802 ± 0.00007	0.21991 ± 0.00008	0.22547 ± 0.00008	0.23317 ± 0.00008	0.23836 ± 0.00007
B	-0.20169 ± 0.00433	-0.18812 ± 0.00527	-0.18213 ± 0.00716	-0.17351 ± 0.00730	-0.16644 ± 0.00889
C	1.91789 ± 0.02450	1.94180 ± 0.03034	1.99089 ± 0.03919	2.01418 ± 0.04000	2.08771 ± 0.04433
D	0.12095 ± 0.00310	0.11501 ± 0.00369	0.11359 ± 0.00537	0.10989 ± 0.00548	0.10898 ± 0.00723
E	0.95794 ± 0.00514	0.95591 ± 0.00694	0.96097 ± 0.00785	0.96081 ± 0.00828	0.96786 ± 0.00751
F	6.67478 ± 0.23957	6.37660 ± 0.28415	5.88852 ± 0.32039	5.67540 ± 0.31558	5.09754 ± 0.29246

Table B1. Parameter values of the function (B1), for different spin values. By using these values, it was possible to describe them with a general functional (B2), whose parameters are in Table B2. The different parameters along with the fitting function and residuals, are shown in Fig. B2.

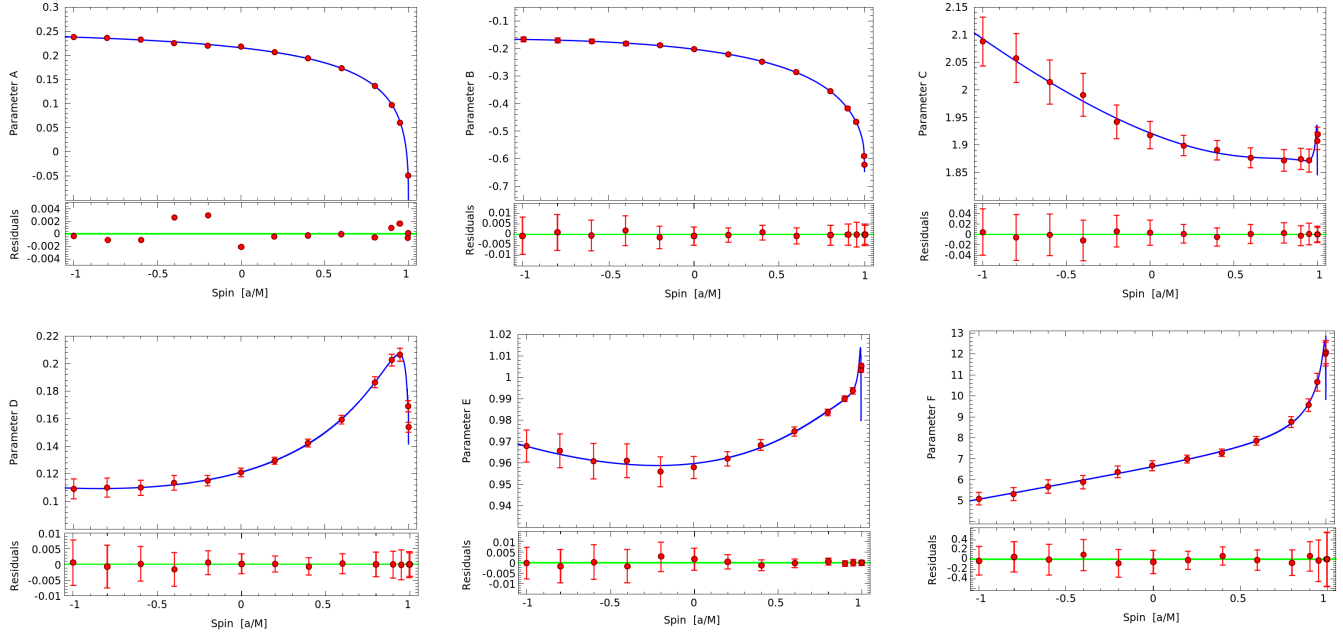


Figure B2. Parameters A , B , C , D , E and F of the expression (19) as a function of the black hole spin a . The fitting function (blue line) has the general form of Eq. (20). Note how the residuals are always of the order of 1% (or less), except for F (of the order of $\sim 10\%$). These relatively large residuals for F , could be reduced using more than 6 parameters for the fit, but we found it unnecessary for the aim of this work. Red bars are associated to the uncertainties on the parameter values from the fit of the bolometric luminosity at different viewing angles (Fig. B1.)

Par of Eq. (20)	α	β	γ	δ	ϵ	ι	κ
Log A	0.21595 ± 0.00065	0.09107 ± 0.00237	-0.05037 ± 0.00657	-0.02739 ± 0.00402	-0.00361 ± 0.00063	0	0
B	-0.20229 ± 0.00039	0.17538 ± 0.00169	-0.14390 ± 0.00657	-0.14534 ± 0.00858	-0.04544 ± 0.00358	-0.00480 ± 0.00045	0
C	1.92161 ± 0.00273	0.27712 ± 0.01259	0.67368 ± 0.04704	0.81327 ± 0.13254	0.48946 ± 0.12580	0.13591 ± 0.04499	0.01373 ± 0.00527
D	0.12120 ± 0.00024	-0.07852 ± 0.00110	0.08995 ± 0.00432	0.12746 ± 0.00569	0.04556 ± 0.00238	0.00510 ± 0.00030	0
E	0.95973 ± 0.00080	-0.02003 ± 0.00288	0.09341 ± 0.01218	0.16775 ± 0.02872	0.11440 ± 0.02475	0.03367 ± 0.00842	0.00351 ± 0.00096
F	6.62190 ± 0.02892	-3.84845 ± 0.09932	-3.11662 ± 0.41485	-3.61394 ± 0.60905	-1.54083 ± 0.26548	-0.19834 ± 0.03454	0

Table B2. Parameter values of expression (20) [or equivalently (B2)]: this latter expression describes the dependence on the black hole spin of the Kerr disk bolometric luminosity (19) [or equivalently (B1)].